

The development of research in Linear Algebra in Spain

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The purpose of this article is to describe the genesis and development in Spain over the last thirty years of a community of researchers in the areas of Linear Algebra and Matrix Analysis.

To put things in a historical context, the 1970s were a period of deep transformation for Spanish universities: research in Mathematics had been a mostly marginal activity for Spanish scholars in the first half of the 20th century. Only for a short period in the 1920s-30s had there been a serious effort, championed by a group of young mathematicians around Julio Rey Pastor¹, to bring Spanish Mathematics up-to-date and into line with current developments elsewhere in the world. The Civil War destroyed any chance of success for this program and sent a whole generation of well-trained mathematicians into exile, mostly to Latin-America. Research in Mathematics became again the exception, instead of the rule. By the late 1960s, however, Franco's dictatorial regime began to slowly open up to influence from abroad, pushed by the economic boom due, in part, to tourism at an industrial scale as one of the country's main sources of income. This relative openness and economic affluence began to change the ways of Spanish universities: for the first time in decades, funds were allocated to promote scientific research, international connections were slowly established, fellowships were granted to young undergraduates to do their PhDs abroad, and Spanish Mathematics began its long journey to its integration into the scientific world at large. At the same time, the economic boom created a new middle class, who expected their offspring to go to university. Consequently, traditional universities had to grow fast in order to accommodate this new inflow of students (up to that point, university had been a prerogative of the upper crust of Spanish society).

On top of this, once Franco died in 1975, and after the consequent political turmoil, the new democratic system chose a de-centralized federal-like territorial organization, shifting part of the political power to the regions. This brought the creation, in the 1980s and, especially, in the 1990s, of several new universities, promoted by the regions and intended to attract its students within the region itself, instead of the handful of larger old, traditional universities, which were supposed to attract students from all over Spain. Both the growth of traditional universities and the creation of new ones brought the need of staffing them with properly trained faculty, able not only to teach, but also to do research. It is in this transitional period between the old and the new system of higher education that our story begins to unfold.

The first three places in Spain where research groups in Linear Algebra were formally established were (in this order) Vitoria, Valencia and Barcelona, all three in the 1980s.

¹see [3] for a thorough investigation of the importance of Rey Pastor in bringing key linear algebraic concepts into the Spanish mathematics university curriculum.

The birth of the basque group at Universidad del País Vasco (UPV-EHU) in Vitoria is inseparable from Graciano de Oliveira and his group at the University of Coimbra: the first contact took place when Juan Miguel Gracia, who was at the time at Colegio Universitario de Álava (UPV-EHU), met José Vitória, a colleague of Graciano at the Coimbra Math Department, while attending the 7th Spanish-Portuguese Mathematical Meeting at Sant Feliú de Guíxols in May 1980. Juan Miguel had become interested in Matrix Analysis via his work on differential equations, and grabbed the opportunity to take advantage of the portuguese colleagues' wide expertise in the subject. This initial contact led to a visit of Juan Miguel to Coimbra in september 1981, which resulted in Graciano committing himself to act as a sort of long-distance mentor for a Linear Algebra group to be created in Vitoria (see de Oliveira's own account in [40]). The initial core of this group, which amounted basically to Juan Miguel and his student, Ion Zaballa², would soon be enlarged, so that by the end of 1982 a weekly group seminar was taking place every Friday in Vitoria, and members of both groups in Coimbra and Vitoria began to exchange short visits on a regular basis.

From that point on, a series of meetings were held in order to regularly bring together spanish and portuguese linear algebraists: three consecutive editions took place in 1982, 1983 and 1984 in Coimbra, Vitoria and Coimbra, respectively. Seen in retrospect, the 1983 meeting in Vitoria was one of the key moments in the creation of the spanish Linear Algebra community: besides bringing well-established invited speakers from abroad, such as Stephen Barnett, Avi Berman, Charlie Johnson or Bob Plemmons, the so-called *International Meeting on Linear Algebra and Applications*, hosted by the Vitoria group, attracted numerous spanish researchers, coming from widely differing backgrounds, which would become the seed for today's Linear Algebra community. Among them, for instance, was Rafael Bru, from Universitat Politècnica de Valencia (UPV), who got interested in the intricacies of the Jordan canonical form by Manuel López Pellicer, a functional analyst who was his colleague at UPV's Mathematics Department. Also attending was Vicente Hernández, who would co-lead with Bru the UPV group during its first years, until he moved away from Linear Algebra into Computer Science in the late 1990s. Encouraged by what they saw at the meeting, Bru and Hernández formally created a research group at UPV in a year's time, by 1984-85. This group would in time become the most numerous in Spain, with its scientific offspring spreading to the newly created universities in the east of Spain.

The Vitoria 1983 meeting also helped to create a close scientific relationship between the Vitoria group and members of Universitat Politècnica de Catalunya (UPC) at Barcelona. The leading figure among the latter was Ferrán Puerta. His pathway into Linear Algebra came through teaching (he wrote a Linear Algebra textbook [74] in the 1970s, which was among the first ones written in Spain). The background of the Barcelona group, mostly Algebraic and Differential Geometry, brought new approaches and insights into the problems in the

²Graciano de Oliveira would ultimately become Zaballa's PhD advisor

Theory of Linear Control Systems the basque group was working on, but was probably also responsible for the slow build-up of the Barcelona group as an established Linear Algebra research group, which did eventually come about in the late 1980s.

The fourth edition of the Spanish-Portuguese meetings was subsumed into the *IV International Conference on Linear Algebra and Applications*, organized in Valencia by the UPV group in 1987. This was a large international conference, with over two hundred participants from all over the world, which somehow amounted to the first steps of Spain's Linear Algebra's integration into the international scene. Incidentally, let it be said that also the birth of ILAS was in the cooking at the time: the 1987 Valencia conference happened to host a meeting of the international board of IMG/Tilac (the International Matrix Group/The International Linear Algebra Community), which would soon be renamed as the International Linear Algebra Society (see [23] for a very detailed report on this conference).

The 1990s was a period of consolidation for the spanish Linear Algebra community. New groups were created and several meetings, both national and international, were held in Spain. Among the latter, the 1994 Meeting on Total Positivity and its Applications [54], held in Jaca in september 1994, represents a milestone in the creation of a Linear Algebra research group at Universidad de Zaragoza: Mariano Gasca, one of the local organizers, was one of several classical analysts who attended the previously mentioned 1983 Vitoria meeting. He had become drawn to Linear Algebra by his research on the interpolation of functions of several real variables, which led him in turn to the study of totally positive matrices. Juan Manuel Peña, a young collaborator of Gasca at the time, continued this line of research as the leader of a group at Universidad de Zaragoza whose research exploits the connection of such matrices with applied topics.

Another most relevant international meeting was the 8th ILAS Conference, organized by the UPC group in Barcelona in july 1999, which was another landmark in the process of getting the spanish Linear Algebra community closer to their international peers. As to more locally focused meetings, a new series was launched, the so-called *Encuentros de Análisis Matricial - EAMA* (Matrix Analysis Meetings), which were instrumental in strengthening the ties between the emerging groups. Although in principle the EAMAs intended to further the spanish-portuguese connection, portuguese attendance declined over time: three EAMAs were organized, one in Valencia in 1989, another in Vitoria in 1994 and the last one in Sevilla in 1997, with an ILAS Conference in between, held in Lisbon in 1992. By 1994, the spanish attendants to the EAMA in Vitoria outnumbered the portuguese by six to one (84 to 14, see the report [39]).

Also in the 1990s, Vicente Hernández established valuable connections with researchers from all over Europe as his group at UPV joined the effort in building up the NICONET Network on Numerics in Control. This european network would officially start its activities in 1998, funded by the European Community,

and helped triggering a good deal of research in High Performance Computing at UPV and, later on, also at Universitat Jaume I in Castellón (see below).

As it turned out, the 1999 ILAS meeting was the start of almost a decade in which no Linear Algebra meetings would be held in Spain at all. By the mid 2000s it became clear that, although some of us would still meet occasionally at Linear Algebra conferences abroad, or at the biennial CEDYA meetings of the Spanish Applied Mathematics Society (SEMA), the lack of some kind of regular gatherings for our community was thwarting possible collaborations, as well as any other kind of interactions, between groups. Once a consensus was met that some sort of stable association would be advisable, the 2007 CEDYA meeting was chosen as the setting of a formal gathering, where members of the spanish Linear Algebra research groups were summoned to agree on the terms of such an association. The outcome of this meeting was the decision to apply for official endorsement from the spanish Ministry of Science as a Thematic Network, an administrative status bringing not only endorsement as a recognized scientific community, but also a small yearly funding to subsidize the basic activities of the network. Thus the Network ALAMA (an acronym, in spanish, for *Linear Algebra, Matrix Analysis and Applications*) was officially born when the application succeeded in 2008. So far, ALAMA has been extremely useful as an organizational umbrella under which several activities transversal to the groups are coordinated: central among these are, of course, the ALAMA Meetings. They are being held on even-numbered years since 2008, and provide an opportunity for all the groups to get together, interacting with each other and with researchers from abroad. More recently, ALAMA Spring Courses, monographically devoted to some specific topic, are also being organized on odd-numbered years at the CIEM (International Center for Mathematics Meetings) in Castro-Urdiales. So far, two of them have been held, one on Piece-wise Linear Systems in 2012 and one on Matrix Polynomials in 2013. A Spring Course on Totally Positive Matrices is scheduled for 2015. News on these, and other activities, are reported through the network website

<http://www.red-alama.es>

which spreads news and notices on the activities of the network and, more generally, facilitates the exchange of information of any sort among network members. Although government funding has been discontinued since 2010 due to budgetary cuts, the network has continued supporting itself and its activities via registration fees and, especially, the generosity and hard work of network members.

So, what is the Linear Algebra/Matrix Analysis research landscape in Spain as of today? The ALAMA network has over 100 members, some of which might be called just ‘sympathetic by-standers’, (members of other communities who just want to stay aware of developments in ours). There is, however, a core of 60 to 70 members who regularly attend the ALAMA meetings and, occasionally, the Spring Courses. The members of this core belong to research groups based

at over fifteen universities all over Spain. A brief review of the most relevant groups is as follows³:

Starting with foundational ones, the basque group has expanded since the 1980s from Vitoria to Bilbao and San Sebastián, reaching all three campuses of UPV-EHU. Inmaculada de Hoyos, Silvia Marcaida, Alicia Roca and Francisco Velasco, among others, have joined Gracia and Zaballa in the effort of analyzing both Linear Control [78, 57, 4] and matrix perturbation and completion problems [8, 5, 58], with recent incursions into inverse matrix polynomial problems [65]. They have kept their close ties with the Barcelona group at UPC, which has devoted its efforts over the years to the analysis of problems in Mathematical Systems Theory via geometric techniques [49, 37, 7]. Ferrán Puerta, Josep Ferrer and María Isabel García Planas, together with younger researchers such as Albert Compta, Dolores Magret, Marta Peña or Xavier Puerta, have both unveiled and exploited normal and canonical forms for different kinds of systems, as well as for parametrized families of them [51, 50], skilfully using Arnold's techniques on versal deformations [48, 52]. Quite recently, another research group with connections to Linear Algebra has surfaced at UPC, namely the MAPTHE group on M-Matrix Analysis and Potential Theory, some of whose members are Andrés Encinas, Margarida Mitjana or Ángeles Carmona: although their scientific background is mainly Potential Theory, their study of discrete elliptic operators on finite networks has led them down the path to Matrix Analysis [9, 10, 11].

Like the Vitoria group, the one in Valencia underwent a significant expansion in the 1990s to the point of becoming the most numerous group in Spain in the 2000s. After some initial matrix-theoretic publications [19, 21], the group's research soon diversified into a wider range of problems as new researchers, such as Rafael Cantó, Carmen Coll, Josep Mas, Sergio Romero, Elena Sánchez, Néstor Thomé, Juan Ramón Torregrosa or Ana Urbano joined the group. Their research interests cover, among others, linear control systems, with a recent emphasis on positive ones [16, 22, 6], matrix factorizations for numerical methods [26, 55, 27], preconditioning [18, 20], or mathematical modelling [28, 24, 25]. Still, many core linear algebra topics, such as completion problems, H-matrices or generalized inverses, have been (and are still being) investigated at UPV [47, 63, 17, 60, 66]. Also, a standing scientific collaboration is held with Josep Gelonch from Universitat de Lleida.

On a different note, former UPV students of Vicente Hernández have kept working on Linear Algebra from a computational point of view: such is the case, for instance, of José Román, also from UPV, who is the lead developer of SLEPc, the Scalable Library for Eigenvalue Problem Computations⁴, a software library for the solution of large scale sparse eigenvalue problems on parallel computers.

³Although some universities host several separate research groups, I choose not to itemize them for the sake of brevity. Also, only a short sample of (hopefully) representative references are cited for each group. I apologize in advance for any significant omission

⁴see <http://www.grycap.upv.es/slep/>

In Castellón, Enrique Quintana, also a former student of Vicente Hernández, is the group leader of the High Performance Computing and Architectures (HPCA) Group at Universitat Jaume I. Although their main goal is the optimization of (mainly parallel) numerical algorithms, several of these algorithms are meant to solve classical Linear Algebra problems, which requires performing detailed analyses via Linear Algebra techniques in order to fine-tune the algorithms [75, 76, 73].

Other young universities, created in the 1990s in the Valencia region like Universitat Jaume I, were also populated by scientific offspring of the UPV group: Joan-Josep Climent, a former student of Bru, established a group at Universitat d'Alacant whose research, although focused mainly on Cryptology and Coding Theory, has a strong connection with Linear Algebra and linear control systems [34, 35]. Carmen Perea, from Universidad Miguel Hernández in Orihuela, is part of that group as well.

Another group with connections to Numerical Analysis is the one already mentioned at Universidad de Zaragoza, led by Juan Manuel Peña. Their research exploits the connection of totally positive matrices with Neville elimination, eigenvalue localization and CAGD⁵ [53, 72, 38]. The Zaragoza group keeps close ties with groups at the universities of Alcalá and Oviedo: Pedro Alonso, a former student of Peña, leads a group at Universidad de Oviedo conducting research on Numerical Linear Algebra and High Performance Computing for problems of large dimension [1, 2], while José-Javier Martínez and Ana Marco at Universidad de Alcalá are active in exploiting classic analytic tools to devise fast and accurate algorithms for highly structured matrices [67, 68].

Another group in the Madrid area, led by Nieves Castro, is based at Universidad Politécnica de Madrid, and its main research interest lies in generalized inverses [31, 32, 33]. Other groups based in Madrid were somehow triggered by the EAMA meeting held at Vitoria in 1994: that was, for instance, my first personal contact with the Spanish Linear Algebra crowd, and there I met Alberto Borobia. He and I had obtained our undergraduate degree together years ago from Universidad Complutense de Madrid, but had got PhDs in very different areas. During EAMA 1994, however, we were able to find some common ground in the spectral analysis of nonnegative matrices [14], which would later lead to a long collaboration in the nonnegative inverse eigenvalue problem (see [15] and references therein). Borobia and Roberto Canogar at UNED (Spain's public Open University) conduct research on several matrix completion problems as well [12, 13]. EAMA 1994 also gave me the chance of first meeting Froilán M. Dopico and Francisco Marcellán of Universidad Carlos III de Madrid (UC3M). Marcellán is one of the sympathetic by-standers I mentioned above. Like Gasca, he had attended the 1983 Vitoria meeting, and was well aware of the manifold connections of Linear Algebra with classical Analysis. Although we did not talk math at the time, two years later they would both become my next-door colleagues as I was hired by the Math Department in UC3M right after a post-doc

⁵Computer-Assisted Geometric Design

abroad, in which I became interested in eigenvalue perturbation theory [70]. Some initial papers on perturbation theory and high relative accuracy eigenvalue algorithms [71, 44] were followed by others, increasingly concerned with the influence of matrix structure on the properties and behavior of numerical algorithms [45, 64]. Right now there is an ever growing and extremely active group at UC3M, led by Dopico and with Juan Manuel Molera and Fernando de Terán as senior members, doing research in Numerical Linear Algebra with strong matrix-theoretic foundations [41, 42, 43, 46].

Last but not least, Linear Algebra also flourished in Castilla: José Ángel Hermida, who is now at Universidad de León, got his PhD at Universidad de Valladolid in Abstract Algebra under Tomás Sánchez Giralda. One of Sánchez Giralda's research interests, which he passed on to Hermida, was the study of linear systems and, more generally, of matrices with entries belonging to a ring, an area with wide-ranging applications in Control Theory. Nowadays there is a young and active group at Universidad de León, which counts Miguel Carriegos, Montserrat López and Andrés Sáez-Schwedt among its members, exploring this kind of topics [59, 29, 30]. Back at Universidad de Valladolid, Carlos Marijuán and co-authors have been working for years on the nonnegative inverse eigenvalue problem (NIEP): Marijuán's previous research had been focused on Algebraic Graph Theory, and the connection between the spectral properties of nonnegative matrices and the cyclic structure of their associated digraphs is what brought him to the NIEP. One of their most remarkable feats has been solving in 1997 the case $n = 4$ of the NIEP using graph-theoretic techniques[77], but Marijuán and Miriam Pisonero have also been working lately on several other Matrix Analysis problems as well [61, 62].

If we compare this rough recount with the census made in [56] in 1992, it becomes clear that we have come a long way since then: several new groups have emerged, and most of the ones which were active then remain active today. The ALAMA Network has been most useful as a stable platform to coordinate collective activities, some of them in collaboration with similar networks from abroad⁶. International connections have been strong for some time: Spain was, for instance, the country where the First SIAG/Linear Algebra Summer School was held, in July 2008 at the CIEM in Castro-Urdiales. This was just the forerunner of the present widely celebrated Gene Golub SIAM Summer Schools. Furthermore, the 11th SIAM Conference on Applied Linear Algebra, the second one to be held outside of the US, took place in Valencia in June 2012, organized by the group at UPV. Most Spanish researchers in Linear Algebra are members of some international society, such as the ILAS or SIAM (mostly through SIAG-LA), and some of them have served as members of their boards. Although our presence in the editorial boards of the journals in the area is not strong yet, I think we can safely say that our efforts to fully integrate ourselves into the international Linear Algebra community at large have succeeded. Unlike the

⁶The last ALAMA Meeting, held in July 2014 in Barcelona, was organized jointly with the annual workshop of the ANLA Group on Applied and Numerical Linear Algebra of GAMM, the Gesellschaft für Angewandte Mathematik und Mechanik

pioneers in the 1980s and 90s, young PhD students are being trained nowadays in constant connection with the outside world, and are enjoying all the benefits of being part of strong, established research groups. Although prospective PhD students are hard to come by in our country these days, I'm hopeful for the future. I am convinced that both present members of our community, and the younger generations of researchers still to come, will manage to keep Linear Algebra's torch burning in Spain for at least another thirty years

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